Superconformal field theories in various dimensions

1. statistics of students
(# undergrads, #graduate students, ...)
who is interested in PhD in string theory?

2. Why conformal symmetry?
In physics conformal symmetry describes scale invariance of a quantum field theory
cutoff $\Lambda \longrightarrow \underset{\substack{\uparrow \\ \text{scale factor}}}{\lambda} \Lambda$

$\beta(\Lambda) \neq 0 \Rightarrow$ QFT is not scale-invariant
generic situation
but: at low energies a cut-off dependent theory might flow to an RG fixed-point
i.e. $\beta = 0$

$$\left(\text{Recall: } \Lambda \frac{d}{d\Lambda} \underset{\substack{\uparrow \\ \text{coupling constant}}}{g_i}(\Lambda) = \beta_i(g(\Lambda)) \right)$$

$\rightarrow$ effective QFT at low energies that is exactly scale invariant.
trivial cases: free massless theories
interesting cases: interacting conformal field theories

3. Why supersymmetry?

Exact analysis in the infrared possible

→ must exhibit conformal symmetry <u>and</u> supersymmetry

→ such algebras are very constrained and are called "superconformal algebras"

<u>only</u> possible in spacetime dimensions

$$d \leq 6$$

<u>Examples</u>: in string theory we have the world-volume theories of M_2, M_5 and D_3 branes (many other examples ...)

<u>In these lectures</u>:

Will look at SCFT's in

first part ↑	6	↓ second part	(compactification)
	5		
	4		
	3		
	2		

dimensions.

<u>First question</u>: Why only the above dimensions?

§1. The superconformal algebra

Before attempting to understand the superconformal algebra, we first need to look at some preliminaries of the conformal algebra...

§1.1 Conformal field theories

What is a conformal transformation?

A transformation $x^\mu \mapsto \tilde{x}^\mu$ such that

$$g_{\mu\nu}(x) \longmapsto \lambda\, g_{\mu\nu}(\tilde{x})$$

"angle preserving" transformation

The group of such transformations is called the "conformal symmetry group" and is an extension of the Poincaré group.

Its generators are given by:
(Greek indices run from 0 to d-1)

$M_{\mu\nu} = i(x_\mu \partial_\nu - x_\nu \partial_\mu)$ Lorentz generators

$P_\mu = -i\partial_\mu$ translations

$D = (-i)[-x\cdot\partial]$ dilatations

$K_\mu = (-i)[-2x_\mu(x\cdot\partial) + x^2\partial_\mu]$ special conformal transformations

D is a scalar and K_μ is a covariant vector under Lorentz transformations.

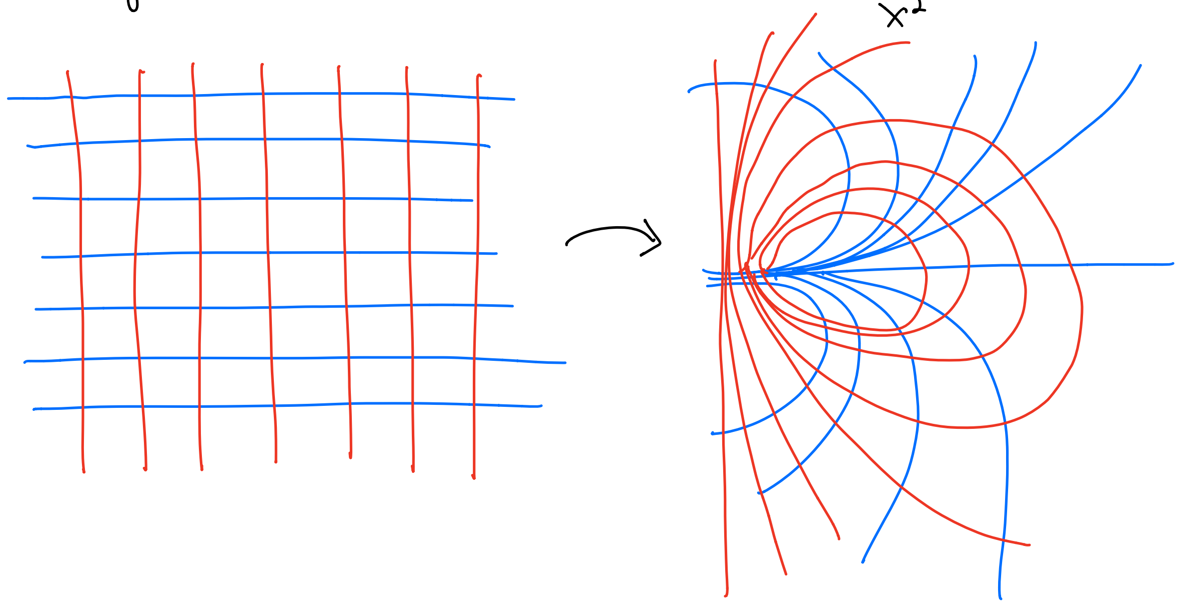
Intuitively, D is obvious because it induces ordinary scale transformations

K_μ are less obvious, they induce so called "Möbius transformations"

$$x^\mu \longmapsto \frac{x^\mu - a^\mu x^2}{1 - 2a\cdot x + a^2 x^2}$$

"inversion + translation + inversion"

by inversion we mean: $x^\mu \longmapsto \frac{x^\mu}{x^2}$



The above generators obey the commutation relations :

$$[M_{\mu\nu}, M_{\alpha\beta}] = (-i)\left[\eta_{\mu\beta} M_{\nu\alpha} + \eta_{\nu\alpha} M_{\mu\beta} - \eta_{\mu\alpha} M_{\nu\beta} - \eta_{\nu\beta} M_{\mu\alpha}\right]$$

$$[M_{\mu\nu}, P_\alpha] = (-i)\left[\eta_{\nu\alpha} P_\mu - \eta_{\mu\alpha} P_\nu\right]$$

$$[D, M_{\mu\nu}] = 0$$

$$[M_{\mu\nu}, K_\alpha] = (-i)\left[\eta_{\nu\alpha} K_\mu - \eta_{\mu\alpha} K_\nu\right]$$

$$[D, P_\mu] = -ik_\mu$$

$$[D, K_\mu] = -i(-K_\mu)$$

$$[D, D] = 0$$

$$[P_\mu, P_\nu] = 0$$

$$[P_\mu, K_\nu] = (-i)[2\eta_{\mu\nu} D + 2M_{\mu\nu}]$$

$$[K_\mu, K_\nu] = 0$$

The conformal group is locally isomorphic to $SO(d,2)$. Denote $SO(d,2)$ generators by S_{ab} where latin indices run from -1 to d. Then we have the following correspondence:

$$S_{\mu\nu} = M_{\mu\nu}$$

$$S_{-1d} = D$$

$$S_{\mu -1} = \frac{1}{2}[P_\mu + K_\mu]$$

$$S_{\mu d} = \frac{1}{2}[P_\mu - K_\mu]$$

We can also define a Euclidean conformal algebra:

$$M'_{pq} = S_{pq}$$

$$D' = iS_{-1 0}$$

$$P'_p = [S_{p-1} + iS_{p0}]$$

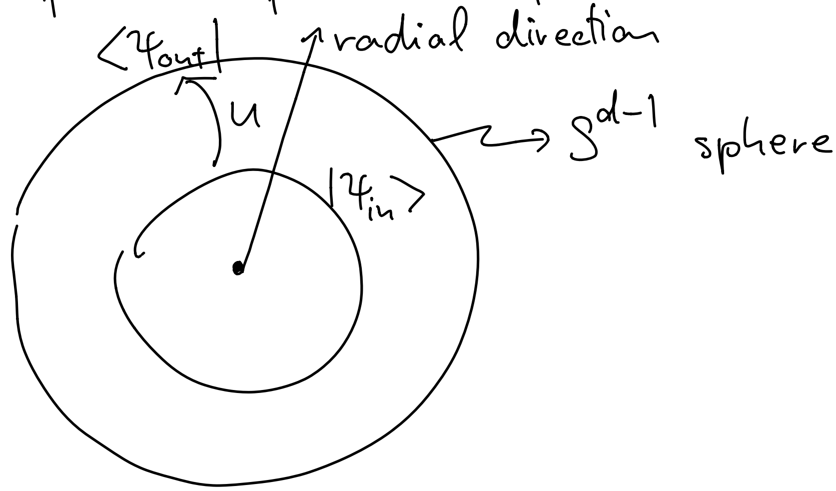
$$K'_p = [S_{p-1} - iS_{p0}]$$

→ generators of Euclidean conformal group $SO(d+1,1)$

Hermiticity properties:

$$M'^{\dagger} = M'$$
$$D'^{\dagger} = -D'$$
$$P'^{\dagger} = K'$$
$$K'^{\dagger} = P'$$

We can understand these properties easily from the point of view of "radial quantization":



where $U = e^{iD\Delta\tau}$, $\tau = \log r$

states are classified according to their scaling dimension

$$D|\Delta\rangle = i\Delta|\Delta\rangle$$

and their $SO(d)$ spin ℓ

$$M_{pq}|\Delta,\ell\rangle_{\{s\}} = (M^R_{pq})_{\{s\}}{}^{\{t\}}|\Delta,\ell\rangle_{\{t\}}$$

since <u>only</u> M commutes with D.

The conjugation operation is given by inversion ($x^\mu \mapsto \frac{x^\mu}{x^2}$) and in-states are taken to live at $x=0$ while out-states are at ∞.

since $[M, D] = 0 \longrightarrow M^\dagger = M$

on the other hand surfaces invariant under P are <u>not</u> equal-time surfaces

$\longrightarrow P^\dagger \neq P$

$$\underset{\text{4}}{IPI} = K = \text{special conformal trsf.}$$